Conductance of Mesoscopic Systems with Magnetic Impurities

M.G. Vavilov and L.I. Glazman

Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455

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We investigate the combined effects of magnetic impurities and applied magnetic field on the interference contribution to the conductance of disordered metals. We show that in a metal with weak spin-orbit interaction, the polarization of impurity spins reduces the rate of electron phase relaxation, thus enhancing the weak localization correction to conductivity. Magnetic field also suppresses thermal fluctuations of magnetic impurities, leading to a recovery of the conductance fluctuations. This recovery occurs regardless the strength of the spin-orbit interaction. We calculate the magnitudes of the weak localization correction and of the mesoscopic conductance fluctuations at an arbitrary level of the spin polarization induced by a magnetic field. Our analytical results for the "h/e" Aharonov-Bohm conductance oscillations in metal rings can be used to extract spin and gyromagnetic factor of magnetic impurities from existing experimental data.

I. INTRODUCTION

Conductance of disordered metals is sensitive to the applied magnetic field. At weak magnetic field the conductivity of a bulk metal has a sharp peak due to the weak localization (WL).[1] Similarly, the conductance of mesoscopic metals fluctuates as the magnetic field is changing.[2,3] Both the conductance fluctuations and the WL correction to the conductivity are quantum mechanical phenomena originating from the interference of quantum states. As any other interference phenomena, they may be demolished by interaction processes.[4]

Localized spins affect the electron transport in metals. Various properties of electron kinetics are sensitive to different aspects of the localized spin dynamics. The energy exchange between electrons in the process of scattering off a magnetic impurity is made possible by the quantum fluctuations of the impurity spin: its virtual flip in the course of scattering facilitates the energy transfer between the two electrons.[5] On the other hand, no spin dynamics of impurities is needed for the suppression of the weak localization correction to conductivity; interaction of electron spins with randomly oriented magnetic moments already leads to that suppression.[6]

Universal conductance fluctuations (UCF) are not suppressed by static magnetic moments. However, even a relatively slow relaxation of individual magnetic moments, such as Korringa relaxation,[7] leads to the time-averaging of the random potential "seen" by transport electrons in the course of measurement, and the mesoscopic fluctuations of the dc conductance get averaged out.[8,9] The sensitivity of conductance fluctuations to the time evolution of the system of localized magnetic moments[10] was used extensively to probe the spin-glass freezing in metals[11,12] and semiconductors.[13]

An applied magnetic field may be used to control the statistical properties and dynamics of localized magnetic moments. It was noticed[12] that in a strong magnetic field the amplitude of conductance fluctuations and "h/e" Aharonov-Bohm (AB) oscillations increases, apparently because the spin fluctuations are quenched.[8] Recently the dependence of the amplitude of "h/e" AB oscillations on

magnetic field was carefully measured on Cu wire rings.[14] The goal of that measurement was to corroborate the existence of localized spins, conjectured on the basis of measurements of the electron energy relaxation rate in Cu wires.[15]

Theory of conductance fluctuations and WL correction to the conductivity at partial spin polarization of magnetic impurities has not been developed yet. Only the limits of no spin polarization at $B = 0$ and of strong polarization at $B \gg T/g\mu_B S$ were considered[8,16–18] (here B is the magnetic field, S is the impurity spin, T is the system temperature, g is the impurity gyromagnetic factor, and μ_B is the Bohr magneton).

In this paper we concentrate on the interference contribution to the linear conductance of mesoscopic systems in the presence of partially polarized magnetic impurities. Particularly, we calculate the weak localization correction to the conductivity and the amplitude of the mesoscopic conductance fluctuations.

II. MAIN RESULTS

It is well known that scattering of electrons off magnetic impurities in the absence of a magnetic field suppresses the interference correction to the conductivity of a wire:[1,6]

$$\Delta\sigma = -\frac{e^2}{2\pi\hbar}\sqrt{D}\left(\frac{3}{\sqrt{2/3\tau_s}} - \frac{1}{\sqrt{2/\tau_s}}\right). \qquad (1)$$

Here D is the diffusion constant for electrons in the wire and $1/\tau_s$ is the electron scattering rate off magnetic impurities. To the lowest order in the exchange constant J calculation, $1/\tau_s = 2\pi\nu n_s J^2 S(S+1)$, where n_s is the concentration of magnetic impurities, ν is the electron density of states at the Fermi level per spin degree of freedom, and S is the magnitude of impurity spins. Taking into account the Kondo renormalization of the exchange constant, at temperatures well above the Kondo

temperature T_K, this rate can be cast in the form

$$\frac{1}{\tau_s} = \frac{8\pi n_s}{\nu} \frac{S(S+1)}{\ln^2 T/T_K}, \quad T \gg T_K. \tag{2}$$

It is clear from Eq. (2) that the phase relaxation rate increases upon reduction of the temperature towards T_K. This rate reaches maximum at $T \sim T_K$, and decreases if the temperature is further reduced. The specific form of function $\tau_s(T)$ at low temperatures depends on the spin S of local moments. The screening of the local spin $S = 1/2$ is complete at $T = 0$ and at $T \ll T_K$ the phase relaxation rate can be found from the Noziéres' Fermi liquid theory,

$$\frac{1}{\tau_s} \propto \frac{n_s}{\nu} \left(\frac{T}{T_K} \right)^2, \quad T \ll T_K. \tag{3a}$$

If $S \geq 1$, then screening is incomplete, and the logarithmic renormalization theory can be employed for the evaluation of τ_s at low temperatures,[19]

$$\frac{1}{\tau_s} = \frac{8\pi n_s}{\nu} \frac{S^2 - 1/4}{\ln^2 T_K/T}, \quad T \ll T_K. \tag{3b}$$

Note, that the low-temperature asymptotes, Eqs. (3), differ from the approximate expression, used sometimes in the analysis of experiments.[20,21]

In this section, we only present the result for WL correction $\Delta\sigma$ to the conductivity of a thin wire in the absence of spin-orbit scattering. If, in addition, the density of magnetic impurities is small, then starting from rather weak fields the condition $g\mu_B B\tau_s \gg 1$ is satisfied, and we find

$$\Delta\sigma = -\frac{e^2}{4\pi\hbar} \int \frac{d\varepsilon/T}{\cosh^2 \varepsilon/2T} \frac{\sqrt{D}}{\sqrt{\Gamma(\varepsilon) + \vartheta DAB^2/\Phi_0^2}}. \tag{4}$$

Here A is the wire cross-section area, ϑ is a numerical factor, and $\Phi_0 = hc/2e$. Function $\Gamma(\varepsilon)$ represents the spin flip rate in the presence of magnetic field B:

$$\Gamma(\varepsilon) = \left[1 - \frac{\langle \hat{S}_z^2 \rangle + \langle \hat{S}_z \rangle \tanh(\varepsilon + g\mu_B B)/2T}{S(S+1)} \right] \frac{1}{\tau_s}, \tag{5}$$

Here $\langle \ldots \rangle$ stands for the thermodynamic average over the states of an isolated impurity spin, see Eq. (15). The Kondo-renormalized rate $1/\tau_s$ in Eq. (5) is given by Eq. (2) at weak magnetic field, $B < T/g\mu_B$, and by

$$\frac{1}{\tau_s} = \frac{2\pi n_s}{\nu} \frac{S(S+1)}{\ln^2[g\mu_B B/T_K]} \tag{6}$$

at strong magnetic field, $B > \max\{T, T_K\}/g\mu_B$.

In the domain of relatively weak magnetic fields, $1/g\mu_B\tau_s \ll B \ll T/g\mu_B$, the spins are not polarized, however, their precession already affects the value of WL correction. Equation (4), valid in this domain, yields $\Delta\sigma$ differing from the full zero-field value (1) by a factor

$2/(3 - \sqrt{1/3}) \approx 0.83$. We discuss the detailed behavior of the WL correction in the field region $B \lesssim 1/g\mu_B\tau_s$ in Section IV. The main variation of the WL correction, however, is associated with the spin polarization. This variation occurs at $B \sim T/g\mu_B$, and is described by Eqs. (4) and (5).

At strong magnetic field, $B \gtrsim T/g\mu_B$, the spin flip rate of thermal electrons ($\varepsilon \sim T$) becomes exponentially small, $\Gamma \propto \exp(-g\mu_B B/T)/[(S + 1)\tau_s]$. In this case higher-order terms in the exchange interaction must be retained to calculate the phase relaxation rate. We find the following result for $\Gamma(\varepsilon)$:

$$\Gamma(\varepsilon) = \frac{1}{2\pi} \frac{\nu}{n_s \tau_s^2} \frac{\pi^2 T^2 + \varepsilon^2}{[g\mu_B B(S+1)]^2}. \tag{7}$$

The crossover between the rates given by Eqs. (5) and (7) occurs at

$$B_*(T) \approx (T/g\mu_B) \ln(\nu/Sn_s\tau_s). \tag{8}$$

The relaxation rate at high fields, Eq. (7), may still be large enough to compete with the conventional mechanisms[1,4] of phase relaxation caused by the electron-electron and electron-phonon interactions.

If one disregards the orbital effect of magnetic field (very thin wire) then the WL correction to the conductivity, Eq. (4), is a function of the ratio B/T, up to slowly varying logarithmic factors $\ln(g\mu_B B/T_K)$ and $\ln(T/T_K)$, see Eqs. (2) and (6).

Different regimes for the WL are schematically represented in Fig. 1. Substantial variation of $\Delta\sigma$ with B occurs at $B \gtrsim T/g\mu_B S$. The curve $B = B_*(T)$ is shown in Fig. 1 by the bold solid line.

The effect of magnetic-field-induced polarization of localized spins on the WL correction to the conductivity is significant only if two quite stringent conditions are met.

First, the orbital effect of the magnetic field B on WL[8] is needed to be small in the range of fields $B \gtrsim T/g\mu_B$, where the spin polarization is significant. The orbital effect dominates over the spin scattering, if $DAB^2/\Phi_0^2 \lesssim \Gamma(\varepsilon \simeq T)$, see Eq. (4). This condition defines the upper bound (a dashed line in Fig. 1) for the domain of fields B in which the spin polarization noticeably affects the WL correction. In general, to avoid the undesirable orbital effect, measurements are to be done on thin wires at low temperature.

Second, the spin-orbit scattering must be sufficiently weak. This condition limits the range of host conductors to materials with sufficiently low atomic numbers.[1] In heavier materials, the spin-orbit interaction suppresses the triplet part of the WL correction to the conductivity, represented by the first term in Eq. (1). Polarization of local moments does not eliminate the randomness of the effective magnetic field induced by spin-orbit interaction, and the triplet contribution to the WL remains suppressed. At the same time, the singlet part of the weak (anti)localization correction [the second term in Eq. (1)] is insensitive to the spin polarization.
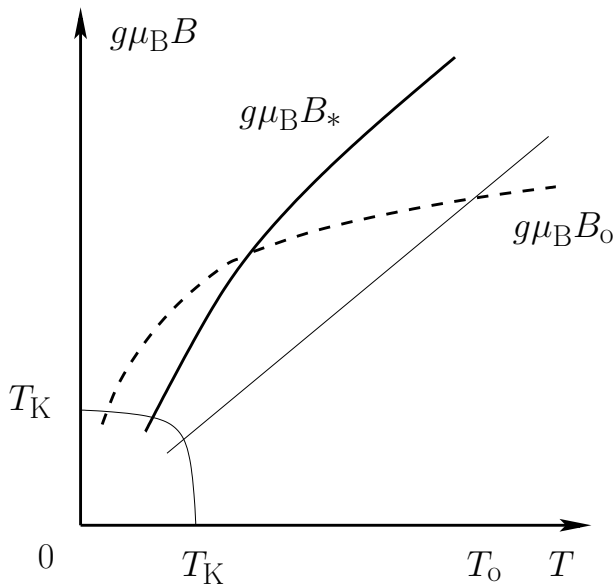
FIG. 1: Different regimes of the temperature, T, and magnetic field, B, dependence of the mesoscopic conductance fluctuations and the WL correction to conductivity. Below the Kondo temperature, $T \lesssim T_K$, and at weak magnetic field, $g\mu_B B \lesssim T_K$, magnetic impurities are screened and the electron phase relaxation rate is given by Eqs. (3). The impurity magnetic moments acquire a significant polarization at fields around $B = T/g\mu_B S$ (straight solid line), which results in strong dependence of the electron phase relaxation rate on the applied magnetic field. The exponential behavior of the phase relaxation rate on B/T is replaced by a weaker power-law function, see Eq. (7), at a crossover field B_* defined by Eq. (8) and depicted by the solid bold curve. Strong variation of the WL correction is possible only in the absence of spin-orbit interaction and at sufficiently weak orbital effect of the magnetic field. This last condition is satisfied below the dashed curve $B_o(T)$ defined by Eq. (34), which intersects the line $g\mu_B SB = T$ at temperature T_o.

The effect of spin polarization on the mesoscopic conductance fluctuations is not subject to the two restrictions discussed above. In other words, the manifestation of the spin polarization in the conductance fluctuations and in the "h/e" Aharonov-Bohm effect is much more robust than that in the WL. The presence of magnetic impurities suppresses the conductance fluctuations only due to the spin dynamics of the impurity system. If the measurement time is significantly longer than the characteristic time of the variation of impurity spin configuration, then the conductance fluctuations are averaged out. Quenching of the spin configuration by the applied magnetic field decreases the effect of the conductance averaging and restores the fluctuations. In the limit of full polarization, magnetic impurities no longer affect the conductance fluctuations.[8]

We concentrate on the amplitude of the Aharonov-Bohm "hc/e" conductance oscillations, since they are exponentially sensitive to the polarization of impurity spins. Magnetic field flux Φ threading the ring of radius R changes electron wave functions and, consequently, the conductance g_Φ of the ring. The conductance statistics is characterized by the correlation function:

$$\langle\langle g_\Phi g_{\Phi+\Delta\Phi}\rangle\rangle = \sum_{k=0}^{\infty} \mathcal{K}_k \cos\left(2\pi k \frac{\Delta\Phi}{\Phi_0}\right), \qquad (9)$$

where $\Phi = \pi R^2 B$ is the magnetic flux through the ring.

We find the amplitude of oscillations of the conductance correlation function:

$$\mathcal{K}_k = \alpha \frac{e^4}{32\hbar^2} \frac{D^{3/2}}{R^3 T^2} \int \frac{e^{-2\pi k R\sqrt{\Gamma(\varepsilon)/D}}}{\sqrt{\Gamma(\varepsilon)}} \frac{d\varepsilon}{\cosh^4 \varepsilon/2T}, (10)$$

where

$$\Gamma(\varepsilon) = \left[1 - \frac{\langle\hat{S}_z\rangle^2 + \langle\hat{S}_z\rangle\tanh(\varepsilon + g\mu_B B)/2T}{S(S+1)}\right] \frac{1}{\tau_s} \tag{11}$$

is the phase relaxation rate to the second order in the exchange interaction J, and α is a dimensionless geometry dependent factor. As the magnetic field increases, $\Gamma(\varepsilon)$ decreases exponentially and at field B_*, given by Eq. (8), the higher order term in the exchange interaction, Eq. (7), prevails.

Equation (10) is valid at $\Gamma(\varepsilon \simeq T) \ll T$. This condition is always satisfied at a sufficiently strong magnetic field, since $\Gamma(\varepsilon \simeq T) \to 0$ as B increases and the impurity spins become almost polarized, see Eqs. (7) and (11). Here again we disregarded other mechanisms of phase relaxation, such as the electron-electron interaction.[1,22]

We emphasize that Eq. (11) contains the average $\langle S_z\rangle^2$ instead of the term $\langle S_z^2\rangle$ in Eq. (5). The two equations are different because of the nature of the electron dynamics producing the WL correction to the conductivity and conductance fluctuations. The difference can be understood in the following way.

The weak localization correction to the conductivity originates from the electron passage along the same trajectory twice. The time difference between the two passages does not exceed the time of phase relaxation, $[\Gamma(\varepsilon \sim T)]^{-1}$ in our case. In the derivation of Eq. (5) we assumed this time being shorter than the Korringa relaxation time[7] τ_T, see Eq. (18). Hence the instantaneous spin-spin correlator $\langle S_z^2\rangle$ enters Eq. (5). The origin of the correlations in mesoscopic conductance is also due to passage of electrons along the same trajectory. Here, however, the relevant time difference is defined by the time between the measurements, and typically greatly exceeds τ_T. That is why the conductance correlation function is described by $\langle S_z\rangle^2$ correlator, characterizing a non-fluctuating component of impurity spins.

Note that in the case of low concentration of magnetic impurities $n_s \lesssim \nu T$ the Korringa relaxation time may become shorter than $[\Gamma(\varepsilon \sim T)]^{-1}$. In this case Eq. (11) rather than Eq. (5) defines the phase relaxation rate in the WL correction to the conductivity. The corresponding modification of the WL correction at $B = 0$ was considered earlier in Ref. [18].

To summarize, we studied the effect of magnetic field on the weak localization correction and on the magnitude of mesoscopic conductance fluctuations in a conductor with magnetic impurities. Our results are valid at an arbitrary level of the spin polarization. We demonstrate that the electron phase relaxation rate acquires energy dependence due to the Zeeman splitting. Such energy dependence is absent in the limits of no impurity polarization, $B = 0$, and strong polarization, $g\mu_B B \gg T$. However, in the experimentally relevant intermediate regime, $g\mu_B B \sim T$, the effect of spin polarization can not be accounted for by assigning a single phase relaxation rate to all electron states.

III. MODEL

We consider a metal with isotropic elastic scattering of electrons by impurities, characterized by the mean scattering rate of electrons $1/\tau_\mathrm{e} = 2\pi\nu n_\mathrm{e} U^2$. Here ν is the electron density of states at the Fermi surface per spin state, n_e is the concentration of impurities, and U is the Born amplitude of elastic scattering by an impurity.

The scattering of conducting electrons off magnetic impurities is described by the Hamiltonian:

$$\hat{H}_\mathrm{m} = J\hat{\vec{S}}\hat{\vec{\sigma}}, \tag{12}$$

where $\hat{\vec{S}}$ is the spin operator of a magnetic impurity, and J is the exchange constant. The electron scattering rate by magnetic impurities is

$$\frac{1}{\tau_\mathrm{s}} = 2\pi\nu n_\mathrm{s} J^2 S(S+1). \tag{13}$$

Here n_s is the magnetic impurity concentration, and S is the total impurity spin. The exchange constant J is renormalized due to the Kondo effect: $J \to 2/(\nu \ln T/T_K)$ at temperatures T exceeding both T_K and $g\mu_\mathrm{B} B$. In stronger fields, $B > \max\{T, T_K\}/g\mu_B$, temperature T under the logarithm is replaced by $g\mu_\mathrm{B} B$.

We study the effect of magnetic impurities on conductivity of metals. In order to evaluate the interference effects in electron transport we will need instantaneous, $\langle S_z \rangle$ and $\langle S_z^2 \rangle$, and time delayed spin-spin correlation functions:

$$\chi_z(\tau) = \frac{\langle \hat{S}_z(t)\hat{S}_z(t-\tau)\rangle}{S(S+1)}, \tag{14a}$$

$$\chi_\perp(\tau) = \frac{\langle [\hat{S}_+(t), \hat{S}_-(t-\tau)]_+\rangle}{S(S+1)}. \tag{14b}$$

Here $\hat{S}_z(t)$ and $\hat{S}_\pm(t) = \hat{S}_x(t) \pm i\hat{S}_y(t)$ are parallel and perpendicular spin components of a magnetic impurity, $[\cdot, \cdot]_+$ is an anticommutator, and

$$\langle \hat{A}\rangle = \frac{1}{Z}\sum_{m=-S}^{S} A_{mm} e^{m\omega_S/T}, \quad Z = \sum_{m=-S}^{S} e^{m\omega_S/T} \tag{15}$$

stands for the thermodynamic average in the presence of a magnetic field B, producing the Zeeman splitting $\omega_S = g\mu_B B$ of the magnetic impurity states; g is the impurity spin gyromagnetic factor.

For a metal with dilute non-interacting magnetic impurities, functions $\chi_z(\tau)$ and $\chi_\perp(\tau)$ have the form:

$$\chi_z(\tau) = \frac{\langle S_z\rangle^2 + \{\langle S_z^2\rangle - \langle S_z\rangle^2\} f_z(\tau)}{S(S+1)}, \tag{16a}$$

$$\chi_\perp(\tau) = \frac{\{S(S+1) - \langle S_z^2\rangle\} f_\perp(\tau)}{S(S+1)}. \tag{16b}$$

The functions $f_z(\tau)$ and $f_\perp(\tau)$ describe spin relaxation. In a dilute system of magnetic impurities, relaxation occurs due to the Korringa mechanism[7] and is exponential:

$$f_\perp(\tau) = e^{-|\tau|/\tau_\perp + i\omega_S\tau}, \qquad f_z(\tau) = e^{-|\tau|/\tau_z}. \tag{17}$$

Here $\tau_\perp$ and τ_z are the spin relaxation times for components perpendicular and parallel to the applied magnetic field, respectively. In a weak magnetic field the two spin relaxation times coincide and are equal to the Korringa time τ_T:

$$\frac{1}{\tau_\mathrm{T}} = \frac{1}{\tau_\perp} = \frac{1}{\tau_z} = \frac{2\pi}{3}(J\nu)^2 T. \tag{18}$$

The applied magnetic field produces Zeeman splitting ε_Z of energy states of transport electrons. When estimating the Zeeman energy ε_Z in metals with magnetic impurities, it is important to notice that the sign of the exchange constant J is fixed by the nature of the pair of host and impurity[13] atoms. Polarization of magnetic impurities results in the exchange contribution $n_\mathrm{s} J\langle S_z\rangle/\mu_\mathrm{B}$ to the effective magnetic field which causes the Zeeman splitting ε_Z of spin states of transport electrons with the gyromagnetic factor g_e

$$\varepsilon_\mathrm{Z} = g_\mathrm{e}\mu_\mathrm{B} B - 2n_\mathrm{s} J\langle S_z\rangle. \tag{19}$$

Equation (19) shows that magnetic impurities may significantly affect the Zeeman splitting of transport electron states.

In the case of antiferromagnetic exchange, $J > 0$, the dependence of ε_Z on B is not monotonic at low temperatures. As we will see below, Eq. (19) provides a mechanism for a non-monotonic in B interference contribution to the metal conductivity, similar to the Jaccarino-Peter mechanism of the reentrant superconductivity.[23,24]

We also consider the effect of spin-orbit scattering on the conduction of electric current. For the specific case of heavy element impurities of concentration n_so, the corresponding term of the Hamiltonian has the form:[25] $\hat{H}_\mathrm{so} = U_\mathrm{so}[\vec{p} \times \vec{p}\,']\hat{\vec{\sigma}}$, and the spin-orbit scattering rate $\gamma_\mathrm{so} = 2\pi\nu n_\mathrm{so} U_\mathrm{so}^2 p_\mathrm{F}^4$.

In this paper we assume that the shortest electron relaxation time is due to elastic scattering and calculate the conductivity to the lowest order in $\tau_\mathrm{e}\gamma_\mathrm{s}$ and $\tau_\mathrm{e}\gamma_\mathrm{so}$, using the standard diagrammatic technique for a disordered

metal.[26] Nevertheless the results are valid even for conductors with strong spin-orbit interaction in their host material, when the spin-orbit scattering rate is comparable with elastic scattering rate.

IV. WEAK LOCALIZATION CORRECTION TO THE CONDUCTIVITY

The weak localization correction to the conductivity is given by the following expression:

$$
\Delta\sigma(\mathbf{B}) = -\frac{e^2 D}{\pi\hbar} \int \frac{d\varepsilon}{4T\cosh^2 \varepsilon/2T}
$$
$$
\times \int \frac{d^d\mathbf{q}}{(2\pi)^d} \sum_{i=1}^{4} \alpha_i \mathcal{C}_i(\varepsilon, \omega = 0, \mathbf{q}), \quad (20)
$$

where the Cooperon $\mathcal{C}(\varepsilon, \omega, \mathbf{q})$ is

$$
\mathcal{C}_i(\varepsilon, \omega, \mathbf{q}) = \frac{1}{i\omega + D\mathbf{q}^2 + \Gamma_i(\varepsilon)}. \quad (21)
$$

The Cooperon components, corresponding to different indices i, represent possible spin configurations: indices $i = 1, 2$ are assigned to the Cooperon components with non-zero spin projections on the direction of magnetic field, $m_c = \pm 1$ and $S_c = 1$; index $i = 3$ is assigned to the Cooperon with $S_c = 1$ and zero spin projection $m_c = 0$; and index $i = 4$ stands for a singlet component, $S_c = m_c = 0$, see Table I.

Performing integration over momentum $\mathbf{q}$, we obtain a general result in the form

$$
\Delta\sigma = -\frac{e^2}{\pi\hbar} \sum_{i=1}^{4} \alpha_i \mathcal{F}_d(\mathbf{B}, \Gamma_i). \quad (22)
$$

Functions $\mathcal{F}_d(\mathbf{B}, \Gamma)$ have different form depending on the conductor geometry ($d = 2$ for a metal film and $d = 1$ for a wire), while parameters α_i and phase relaxation rates $\Gamma_i(B)$ are determined by details of electron scattering processes. First we present functions $\mathcal{F}_d(\mathbf{B}, \Gamma)$ for different conductor geometries and then discuss the effect of spin flip scattering on the weak localization correction to the conductivity.

A. Geometry dependence

The weak localization correction to the conductivity of a quasi two dimensional metal is determined by the following expression:

$$
\mathcal{F}_2(\mathbf{B}, \Gamma_i) = -\frac{1}{4\pi} \left[\psi\left(\frac{1}{2} + \frac{\Gamma_i + \gamma_{\mathrm{orb}}}{4DeB_\perp/\hbar c}\right) + \ln \frac{4eB_\perp D\tau_{\mathrm{e}}}{\hbar c} \right], \quad (23)
$$

TABLE I: The Cooperon phase relaxation factors for various components corresponding different spin configurations. The components with $i = 1, 2, 3$ represent a spin states with total spin one, $S_C = 1$. These spin configurations form a triplet in an isotropic system. Anisotropy due to the applied magnetic field splits the degeneracy. The $i = 4$ component is a singlet spin configuration with zero total momentum, $S_C = 0$.

i	$|S_C, m_C\rangle$	$\Gamma_i \tau_s$	α_i
1	$S_C = 1, m_C = +1$	$1 - \langle S_z^2 \rangle / S^2$	$+1$
2	$S_C = 1, m_C = -1$	$1 - \langle S_z^2 \rangle / S^2$	$+1$
3	$S_C = 1, m_C = 0$	$2\langle S_z^2 \rangle / S^2$	$+1$
4	$S_C = 0, m_C = 0$	2	-1

where $\psi(x)$ is the digamma function and Γ_i is the dephasing rate for the ith component of the Cooperon. Here Γ_i includes dephasing γ_{orb} by the applied magnetic field parallel to the film $\gamma_{\mathrm{orb}} = e^2 B_\parallel^2 D a^2 / 12\hbar^2 c^2$, where $B_\perp$ and $B_\parallel$ are the magnetic field components, perpendicular and parallel to the film. In weak perpendicular magnetic field we use $\psi(1/2 + x) \approx \ln x + 1/24x^2$ and obtain:

$$
\mathcal{F}_2(\mathbf{B}, \Gamma_i) = \frac{1}{4\pi} \left[\ln \frac{1}{(\Gamma_i + \gamma_{\mathrm{orb}})\tau_{\mathrm{e}}} - \frac{2(DeB_\perp/\hbar c)^2}{3(\Gamma_i + \gamma_{\mathrm{orb}})^2} \right]. \quad (24)
$$

The terms with τ_{e} in Eqs. (23) and (24) originate from the ultraviolet logarithmic divergence of the momentum integral for the weak localization correction to the conductivity in two dimensional case. The existence of this term complicates analysis of the magnetoresistance measurements.

In the quasi one dimensional case the integral over momentum gives:

$$
\mathcal{F}_1(\mathbf{B}, \Gamma_i) = \frac{1}{2} \sqrt{\frac{D}{\Gamma_i + \gamma_{\mathrm{orb}}}}. \quad (25)
$$

The denominator of Eq. (25) contains the sum of electron phase relaxation rates due to scattering on magnetic impurities and due to the orbital effect of the applied magnetic field γ_{orb}. For a magnetic field of strength B and applied in direction $\mathbf{n}$, the orbital contribution to the phase relaxation rate is

$$
\gamma_{\mathrm{orb}} = \vartheta(\mathbf{n}) \frac{e^2 B^2 DA}{\hbar^2 c^2}, \quad (26)
$$

where the function $\vartheta(\mathbf{n})$ of the direction $\mathbf{n}$ is of the order of unity, and A is the area of the wire cross-section.

In the following subsections we focus on properties of the phase relaxation rate Γ_i due to scattering on magnetic impurities. To be specific, we will present the WL correction to the conductivity of a thin wire. Since the phase relaxation factors Γ_i do not depend on the sample geometry, the results for the WL correction to the conductivity of the next two subsections can be easily
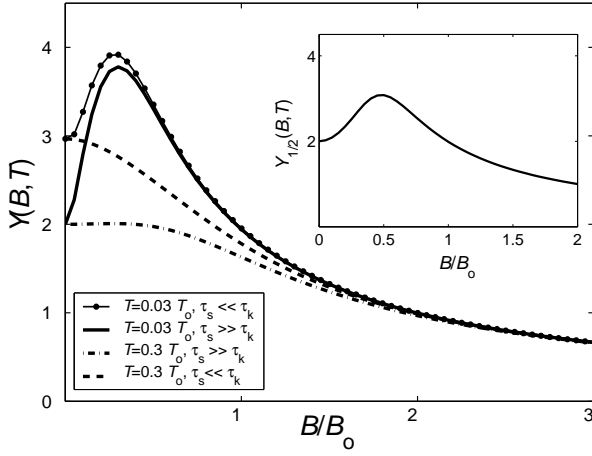
FIG. 2: Dependence of the weak localization correction on the applied magnetic field in a metal with classical spin impurities. The plot shows function $Y(B,T)$ at two temperature values $T = 0.3T_o$ and $T = 0.03T_o$ for both short ($\tau_T \ll \tau_s$) and long ($\tau_K \gg \tau_s$) impurity spin relaxation time τ_T. The subplot shows the WL correction to the conductivity for $S = 1/2$ impurities at $T = T_o$. Here T_o is given by Eq. (35).

generalized to other system geometries, such as a film, a metal ring or an open quantum dot.[27]

B. Effect of classical spin impurities

Even in the absence of spin-orbit interaction, there are five parameters affecting the WL correction to the conductivity, including system temperature T, the applied magnetic field B, impurity spin relaxation time τ_T, the electron scattering rate on magnetic impurities $1/\tau_s$, and the phase relaxation rate due to the orbital effect of the applied magnetic field $\gamma_{\rm orb}$. It is easier to establish the role of these parameters in the case of classical ($S \gg 1$) spins, considered in this Subsection.

The explicit matrix form of the equation for the Cooperon in the case of classical spins is given in Appendix A, see Eqs. (A2) therein. For an arbitrary relation between the electron dephasing rate Γ_i and the impurity spin relaxation rate $1/\tau_T$, the solution of this equation is cumbersome. Here we consider only the limiting cases of long ($\tau_T \Gamma_i \gg 1$) and short ($\tau_T \Gamma_i \ll 1$) spin relaxation time. Usually (for not too small concentration of magnetic impurities $n_s \gtrsim \nu T$) the first condition is satisfied; the second condition may become relevant only at very low n_s. In the first case one may neglect[6] the dynamics of localized spins setting $\tau = 0$ in the correlation functions Eq. (A2b). In the second case, we need to set $\tau \to \infty$ in Eq. (A2b). In both limits we obtain the following expressions for the parameters of Eq. (22)

$$\alpha_{1,2} = 1, \quad \alpha_3 = +\frac{\chi_\perp}{Z\tau_s}, \quad \alpha_4 = -\frac{\chi_\perp}{Z\tau_s} \tag{27}$$

and for the phase relaxation rates

$$\Gamma_{1,2} = (1 - \chi_z)\frac{1}{\tau_s}, \tag{28a}$$

$$\Gamma_3 = (1 + \chi_z)\frac{1}{\tau_s} + i\omega_S - Z, \tag{28b}$$

$$\Gamma_4 = (1 + \chi_z)\frac{1}{\tau_s} + i\omega_S + Z. \tag{28c}$$

Here

$$Z = \sqrt{\frac{\chi_\perp^2}{\tau_s^2} - (\varepsilon_Z + \omega_S)^2};$$

χ_z and $\chi_\perp$ are the impurity spin correlators given by Eqs. (16) and (17) with proper substitution $\tau = 0$ or $\tau \to \infty$. The correlation functions $\langle S_z \rangle$ and $\langle S_z^2 \rangle$ of classical spins are described by the following functions:

$$\langle S_z \rangle = S \coth \frac{S\omega_S}{T} - \frac{T}{\omega_S}, \tag{29a}$$

$$\langle S_z^2 \rangle = S^2 - \frac{2T}{S\omega_S} \coth \frac{S\omega_S}{T} + 2\left(\frac{T}{\omega_S}\right)^2. \tag{29b}$$

The weak localization correction to the conductivity of a wire may be represented as

$$\Delta\sigma_1 = -\frac{e^2}{2\pi\hbar} \sqrt{D\tau_s}\, Y(\mathbf{B}, T). \tag{30}$$

The function $Y(\mathbf{B}, T)$ has different forms, depending on the relations between parameters of the system.

In the limit of long spin relaxation time ($\tau_T \Gamma_i \gg 1$) and weak magnetic field ($\omega_S \tau_s \ll 1$ and $\varepsilon_Z \tau_s \ll 1$) all four Cooperon modes contribute to the weak localization correction to the conductivity

$$Y(\mathbf{B}, T) = \frac{2}{\sqrt{1 - \langle S_z^2 \rangle / S^2 + \gamma_{\rm orb}\tau_s}}$$
$$+ \frac{1}{\sqrt{2\langle S_z^2 \rangle / S^2 + \gamma_{\rm orb}\tau_s}} - \frac{1}{\sqrt{2 + \gamma_{\rm orb}\tau_s}}. \tag{31}$$

The first term in Eq. (31) is due to the Cooperon modes with $i = 1, 2$. It is the first term of the function $Y(B,T)$ which is responsible for a non-monotonic magnetic field dependence of the WL correction to the conductivity. As magnetic field increases and impurity spins become polarized, the phase relaxation rate for modes $i = 1, 2$ decreases, and the first term in Eq. (31) grows. The combination of the remaining two terms, representing Cooperon components $i = 3, 4$, constitutes $0.17\Delta\sigma_1$ of the full value of $\Delta\sigma_1$ at zero magnetic field and monotonically decreases as the magnetic field increases.

We notice that the contribution of $i = 3, 4$ Cooperon components diminishes not only as magnetic impurities become polarized, but also as the Zeeman splitting of conduction electrons and impurity spins ($\varepsilon_Z + \omega_S$) increases, see Eqs. (20) and (27). At strong magnetic

field, $|\omega_S + \varepsilon_Z|\tau_s \gg 1$, the contribution of $i = 3, 4$ Cooperon components to $\Delta\sigma_1$ vanishes, since both the Zeeman splitting of the conduction electrons and the energy transferred in the spin flip process destroy the interference of time reversed paths in the Cooperon. If the spin relaxation time is still long, $\tau_T\Gamma_i \gg 1$, we have

$$Y(\mathbf{B}, T) = \frac{2}{\sqrt{1 - \langle S_z^2 \rangle / S^2 + \gamma_{\mathrm{orb}}\tau_s}}. \qquad (32)$$

For impurity spin system with short spin relaxation time, $\tau_T\Gamma_i \ll 1$, the Cooperon modes with $i = 3, 4$ do not contribute to $\Delta\sigma_1$ again. In this case, the weak localization correction to the conductivity is given by Eq. (30) with

$$Y(\mathbf{B}, T) = \frac{2}{\sqrt{1 - \langle S_z \rangle^2 / S^2 + \gamma_{\mathrm{orb}}\tau_s}}. \qquad (33)$$

We notice that as magnetic field increases, the phase relaxation rate decreases, and the condition of short spin relaxation time ($\tau_T\Gamma_{1,2} \ll 1$) may be reached, even though initially system was in the opposite regime, $\tau_T \gg \tau_s$.

We conclude from Eqs. (31)–(33) that the WL correction to the conductivity depends on the polarization of magnetic impurities through the thermodynamic averages $\langle S_z \rangle$ and $\langle S_z^2 \rangle$. The polarization is significant in an applied field of the order of $B = T/g\mu_B S$, shown as a straight solid line in Fig. 1.

The orbital effect of the applied magnetic field is represented by the term $\gamma_{\mathrm{orb}}\tau_s$ in Eqs. (31)–(33). The orbital effect and the impurity spin polarization compete with each other: the orbital effect suppresses, while the spin polarization enhances the WL correction to the conductivity. The phase relaxation rates γ_{orb} and Γ_1 become equal, $\gamma_{\mathrm{orb}} = \Gamma_1$, at magnetic field $B_o(T)$:

$$B_o(T) = \frac{\Phi_0}{\sqrt{D\tau_s A}}\sqrt{1 - \frac{\langle S_z \rangle_{B=B_o}^2}{S^2}}. \qquad (34)$$

The effect of spin scattering prevails over orbital effect of the magnetic field at $B \lesssim B_o(T)$. On the other hand, the enhancement of the WL correction due to impurity spin polarization is significant at $B \gtrsim T/g\mu_B S$. These two limitations are met if the wire temperature $T \lesssim T_o$, where

$$T_o = g\mu_B S \frac{\Phi_0}{\sqrt{D\tau_s A}}. \qquad (35)$$

At higher temperature, $T \gtrsim T_o$, the effect of the spin polarization is concealed by the orbital effect.

In Fig. 2 we illustrate the effect of an applied magnetic field on the WL correction to the conductivity. We show dependence of the function $Y(B, T)$ on the applied magnetic field B at temperatures $T = 0.3T_o$ and $T = 0.03T_o$. At low temperature $T = 0.03T_o$, the effect of impurity spin polarization is significant, but already at temperature $T = 0.3T_o$ it fades away.

The spin-orbit interaction modifies the WL correction to the conductivity. Using Eqs. (A2) in Appendix A we find:

$$\alpha_{1,2} = 1, \qquad (36a)$$

$$\alpha_3 = \frac{2\gamma_{\mathrm{so}}/3 - \chi_\perp/\tau_s}{\sqrt{(2\gamma_{\mathrm{so}}/3 - \chi_\perp/\tau_s)^2 - \varepsilon_Z^2}}, \qquad (36b)$$

$$\alpha_4 = -\frac{2\gamma_{\mathrm{so}}/3 - \chi_\perp/\tau_s}{\sqrt{(2\gamma_{\mathrm{so}}/3 - \chi_\perp/\tau_s)^2 - \varepsilon_Z^2}} \qquad (36c)$$

and the phase relaxation rates are

$$\Gamma_{1,2} = (1 - \chi_{zz})\frac{1}{\tau_s} + \frac{4\gamma_{\mathrm{so}}}{3}, \qquad (37a)$$

$$\Gamma_3 = \frac{1 + \chi_{zz}}{\tau_s} + \frac{2\gamma_{\mathrm{so}}}{3} + \sqrt{\left|\frac{\chi_\perp}{\tau_s} - \frac{2\gamma_{\mathrm{so}}}{3}\right|^2 - \varepsilon_Z^2}, \quad (37b)$$

$$\Gamma_4 = \frac{1 + \chi_{zz}}{\tau_s} + \frac{2\gamma_{\mathrm{so}}}{3} - \sqrt{\left|\frac{\chi_\perp}{\tau_s} - \frac{2\gamma_{\mathrm{so}}}{3}\right|^2 - \varepsilon_Z^2}. \quad (37c)$$

Here we assumed $\omega_S \ll \Gamma_i$ for simplicity.

According to Eqs. (37), in metals with strong spin-orbit scattering, $\gamma_{\mathrm{so}}\tau_s \gg 1$, the terms with $i = 1, 2, 3$ are suppressed and the interference correction to the conductivity is described by the singlet antilocalization term $i = 4$. In the limit of long impurity spin relaxation time $\tau_T \gg \tau_s$, the weak localization correction to the conductivity depends on τ_s and the Zeeman energy ε_Z:

$$Y(\mathbf{B}, T) = -\frac{1}{\sqrt{2 + 3\varepsilon_Z^2 \tau_s / 4\gamma_{\mathrm{so}} + \gamma_{\mathrm{orb}}\tau_s}}, \qquad (38)$$

while in the opposite limit $\tau_T \ll \tau_s$ the correction also depends on the polarization of impurity spins:

$$Y(\mathbf{B}, T) = -\frac{1}{\sqrt{1 + \langle S_z \rangle^2 / S^2 + 3\varepsilon_Z^2 \tau_s / 4\gamma_{\mathrm{so}} + \gamma_{\mathrm{orb}}\tau_s}}. (39)$$

The terms $\gamma_{\mathrm{orb}}\tau_s \propto A$ in the denominator of Eqs. (38) and (39) originate from the orbital magnetic field effect, see Eq. (26). If the area A is sufficiently small so that the orbital part is not important, and the temperature is low enough, then the antilocalization may show a non-monotonic field dependence due to the Jaccarino-Peter mechanism,[23] see Eq. (19).

From the above analysis performed in the approximation of large impurity spins we make two conclusions. First, the polarization of impurity spins hardly affects the weak (anti)localization correction to the conductivity in metals with strong spin-orbit interaction. Indeed, the only surviving contribution to the conductivity correction originates from the singlet spin configuration of the Cooperon, which is not sensitive to the impurity spin polarization. Second, in metals without spin-orbit interaction, the Cooperon modes with zero spin projection on the magnetic field ($i = 3, 4$) give only a small ($\sim 0.17\Delta\sigma$) contribution to the total WL correction to the conductivity, and their contribution vanishes at moderately strong
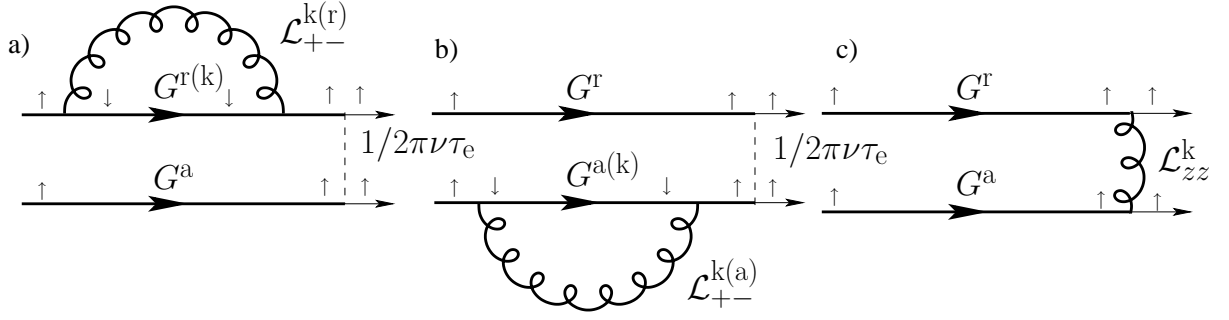
FIG. 3: The Cooperon decay rate is determined by diagrams of the second order in the exchange constant J. The two left diagrams represent the electron self energy contribution from processes with and without (not shown) spin flip. The right diagram represents the vertex correction.

$(\omega_S \tau_{\rm s} \sim 1)$ magnetic field. In the next subsection we neglect these terms and calculate the WL correction to the conductivity, originating from Cooperon modes $i = 1, 2$ at such fields that $\omega_S \tau_{\rm s} \gtrsim 1$. We perform the calculations for an arbitrary value of S.

C. Effect of quantum spin impurities

We notice that the semiclassical description is applicable only *i)* for a large spin $S \gg 1$, when linear in S contributions to the dephasing rates can be neglected and *ii)* at high temperature $T \gg \omega_S$, so that the discreetness of spin energy levels can be disregarded. In this Subsection we consider quantum spins with arbitrary value of S and for arbitrary ratio of ω_S/T. As we discussed in the previous Subsection, only components $i = 1, 2$ of the Cooperon with parallel spins, see Table I, are important at strong magnetic field. The terms with $i = 3, 4$ are small at relatively strong magnetic fields, $|\omega_S + \varepsilon_Z| \tau_s \gtrsim 1$. Moreover, even at weak fields, modes $i = 3, 4$ give rise only to a small portion of $\Delta\sigma_1$, equal to $0.17\Delta\sigma_1$ at zero magnetic field and monotonically decreasing at stronger fields B. To avoid cumbersome expressions, we omit the terms with $i = 3, 4$ in our further analysis.

The contribution to the Cooperon self energy due to the scattering off magnetic impurities is shown in Fig. 3. Details of our calculations are presented in Appendix B. We find that the WL correction to the conductivity may be represented in the form of Eq. (30) with function $Y(\mathbf{B}, T)$ replaced by

$$Y_S(\mathbf{B}, T) = \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2T \cosh^2 \varepsilon/2T} \frac{1}{\sqrt{(\Gamma(\varepsilon) + \gamma_{\rm orb})\tau_{\rm s}}}, \quad (40)$$

where the phase relaxation rate due to scattering off magnetic impurities is

$$\Gamma(\varepsilon) = \left[1 - \frac{\langle \hat{S}_z^2 \rangle + \langle \hat{S}_z \rangle \tanh(\varepsilon + \omega_{\rm s})/2T}{S(S+1)} \right] \frac{1}{\tau_{\rm s}}, \quad (41)$$

provided that the impurity spin relaxation time is large, $\tau_{\rm T} \Gamma(\varepsilon \sim T) \gg 1$.

Spin correlation functions $\langle \hat{S}_z \rangle$ and $\langle \hat{S}_z^2 \rangle$ in Eq. (41) are defined by Eqs. (15). We notice that in strong fields $\omega_S \gtrsim T$ the impurity spins are polarized and the phase relaxation rate of electrons close to the Fermi surface $|\varepsilon| \lesssim T$ vanishes, in agreement with Ref. [8].

We consider the special case of magnetic impurities with $S = 1/2$ in more details. First, we discuss the meaning of the energy-dependent relaxation rate $\Gamma(\varepsilon)$, Eq. (41). In the $S = 1/2$ case, $\Gamma(\varepsilon)$ can be rewritten in the form:

$$\Gamma(\varepsilon) = \frac{4}{3\tau_{\rm s}} \left[p_\downarrow(1 - n(\varepsilon + \omega_S)) + p_\uparrow n(\varepsilon + \omega_S) \right], \quad (42)$$

where $p_{\uparrow(\downarrow)} = (2 \cosh \omega_S/2T)^{-1} \exp(\pm\omega_S/2T)$ is the probability for the spin impurity to be parallel (antiparallel) to the direction of the magnetic field and $n(\varepsilon) = [1 + \exp(\varepsilon/T)]^{-1}$ is the Fermi occupation number for electrons with energy ε at temperature T. We interpret Eq. (42) in the following way. Two processes contribute to the electron phase relaxation rate: *i)* an electron with spin up and energy ε is scattered by an impurity with spin down to the electron state with spin down and energy $\varepsilon + \omega_S$; *ii)* a hole with spin up and energy ε is scattered by an impurity with spin up to the electron state with spin down and energy $\varepsilon + \omega_S$. The probabilities of these processes are determined by the first and second terms in Eq. (42), respectively.

In the limit of short impurity spin relaxation time, $\tau_{\rm T} \Gamma(\varepsilon \simeq T) \gg 1$, the function $Y_{1/2}(\mathbf{B}, T)$ has the form

$$Y_{1/2}(\mathbf{B}, T) \;=\; \int\limits_{-\infty}^{+\infty} \frac{dx}{\cosh^2 x} \left[\frac{2\cosh x}{3\cosh \omega_S/2T \cosh(x + \omega_S/2T)} + \gamma_{\text{orb}}\tau_{\text{s}} \right]^{-1/2}. \tag{43}$$

We illustrate the behavior of function $Y_{1/2}(B, T)$ in the inset of Fig. 2 at temperature $T = 0.3T_{\text{o}}$, where T_{o} is defined by Eq. (35) with $S = 1/2$.

If the orbital effect of the magnetic field is small, $\gamma_{\text{orb}} = 0$, we have

$$Y_{1/2}(\mathbf{B}, T) = \sqrt{\frac{8}{3}} \frac{\sinh(3\omega_S/4T)}{\tanh(\omega_S/2T)\sqrt{\cosh(\omega_S/2T)}}, \tag{44}$$

and the weak localization correction to the conductivity is given by Eq. (30) with $Y_{1/2}(\mathbf{B}, T)$ in the form of Eq. (44). In the opposite limit, when the orbital effect of the magnetic field on WL dominates over the effect of spin scattering, we may expand function $Y_{1/2}(B, T)$ in $(\tau_{\text{s}}\gamma_{\text{orb}})^{-1}$ and obtain

$$\Delta\sigma_1(B) \;=\; -\frac{e^2}{\pi\hbar}\sqrt{\frac{D}{\gamma_{\text{orb}}}}\left(1 - \frac{1}{3}\frac{1}{\tau_{\text{s}}\gamma_{\text{orb}}}\frac{\omega_S}{T\sinh\omega_S/T}\right). \tag{45}$$

We notice that the second term in Eq. (45) has the same structure as the phase relaxation rate suggested in Ref. [18] to describe the WL correction to the conductivity. Therefore, the expansion, presented in Eq. (45), establishes the conditions of applicability of the suggested formula.[18]

Expression for the phase relaxation rate in Eq. (41) was derived in the limit of long impurity spin relaxation time, $\tau_{\text{T}}\Gamma(\varepsilon \sim T) \gg 1$. In the opposite limit, the spins at different moments of time are not correlated, and we have to substitute $\langle S_z \rangle^2$ for $\langle S_z^2 \rangle$ in Eq. (41). Then, the function $Y_{1/2}(\mathbf{B}, T)$ acquires the form:

$$Y_{1/2}(B, T) = \int \left(1 - \frac{1}{3}\tanh^2\frac{\omega_S}{2T} - \frac{2}{3}\tanh\left(x + \frac{\omega_S}{2T}\right)\tanh\frac{\omega_S}{2T} + \gamma_{\text{orb}}\tau_{\text{s}}\right)^{-1/2}\frac{dx}{\cosh^2 x}. \tag{46}$$

We observe from Eq. (41), that at strong magnetic field the phase relaxation rate exponentially vanishes. Consequently, higher order terms in the exchange constant J may become important. The fourth order diagrams for the Cooperon phase relaxation rate are shown in Fig. 4 and the corresponding analytical calculations, presented in Appendix C, yield

$$\Gamma(\varepsilon) = \frac{1}{2\pi}\frac{\nu}{n_{\text{s}}\tau_{\text{s}}}\frac{\pi^2 T^2 + \varepsilon^2}{\omega_S^2(S+1)^2}\frac{1}{\tau_{\text{s}}}. \tag{47}$$

We emphasize that Eq. (47) represents the J^4 contribution to the electron phase relaxation rate, while Eq. (41) represents the J^2 contribution. The corresponding small parameter of the expansion in powers of J^2 may be written as $\nu/(n_{\text{s}}\tau_{\text{s}})$.

As the applied magnetic field increases, both the J^2 and J^4 contributions may become comparable, since the J^2 contribution decreases exponentially, while the J^4 contribution at energy $|\varepsilon| \sim T$ decreases much slower, cf. Eqs. (41) and (47). The small factor $\nu/(n_{\text{s}}\tau_{\text{s}})$ is compensated at strong enough magnetic field B_*, which we

estimate from $\exp(-\omega_S/T) = \nu/(n_{\text{s}}\tau_{\text{s}})$. We obtain

$$B_*(T) = \frac{T}{g\mu_{\text{B}}}\ln\frac{\nu}{n_{\text{s}}\tau_{\text{s}}}. \tag{48}$$

At small fields, $B \lesssim B_*$, the relaxation rate decreases exponentially with the increase of B, whereas at higher fields this dependence is replaced by a slower power law $\Gamma(\varepsilon \sim T) \propto T^2/\omega_S^2$. The counterpart of Eq. (45) at $B > B_*$ reads

$$\Delta\sigma_1(B) \;=\; -\frac{1}{\pi^2}\frac{e^2}{\hbar c}\sqrt{\frac{D}{\gamma_{\text{orb}}}}$$
$$\times \left(1 - \frac{\pi}{3}\frac{\nu}{n_{\text{s}}\tau_{\text{s}}}\frac{1}{\tau_{\text{s}}\gamma_{\text{orb}}}\frac{T^2}{(S+1)^2\omega_S^2}\right). \tag{49}$$

V. CONDUCTANCE FLUCTUATIONS

In this Section we study the effect of the impurity spin polarization on conductance fluctuations of a metal ring, weakly connected to the leads.[28] Magnetic flux Φ piercing the ring changes wave functions of electrons in the
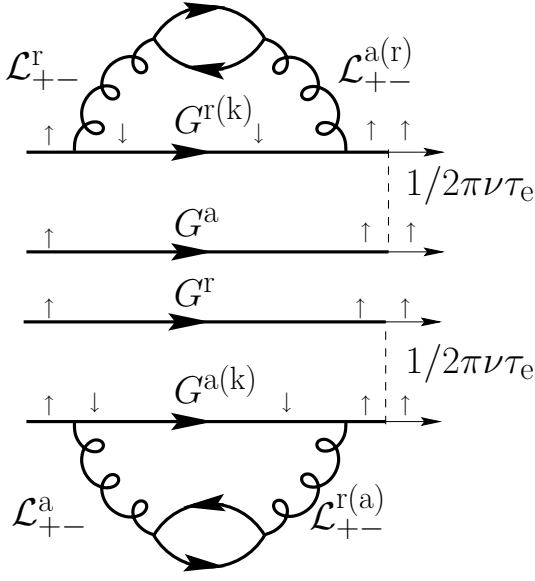
FIG. 4: The Cooperon self energy diagrams to the fourth order in the exchange constant J. The spin-spin correlation function (wavy line) acquires correction due to the electron scattering off the spin.

ring and, consequently, the ring conductance. The fluctuations of the conductance are usually characterized by the correlation function $\mathcal{K}(\Delta t, \Delta\Phi)$:

$$\mathcal{K}(\Delta t, \Delta\Phi) = \langle\langle g_\Phi(0) g_{\Phi+\Delta\Phi}(\Delta t)\rangle\rangle_\Phi, \qquad (50)$$

where $\Phi = \pi R^2 H$ is the magnetic flux through the ring of radius R, and Δt is the time lapse between the measurements of g_Φ and $g_{\Phi+\Delta\Phi}$. The d.c. conductance, Eq. (50), is defined in terms of the current $I(t)$, averaged over measurement time T_m

$$g_\Phi(t) = \int_t^{t+T_m} \frac{\partial I_\Phi(t)}{\partial V}\bigg|_{V=0} dt', \qquad (51)$$

where V is the applied bias. During the measurement time T_m the magnetic flux should be constant, otherwise the measured conductance is already averaged over different realizations.

We assume that the system satisfies the ergodic hypothesis and the averaging over the magnetic field is equivalent to averaging over impurity configurations.[2,29] To calculate the conductance correlation function, Eq. (50), we apply the conventional averaging technique.[26] We consider only the case of short relaxation time $\tau_T \ll \Delta t$. In this case the spin correlation functions in Eqs. (16) should be taken in the limit $\tau \to \infty$, so that $\chi_\perp(\tau \to \infty) = 0$ and $\chi_z(\tau \to \infty) = \langle S_z\rangle^2$.

The conductance correlation function contains all harmonics

$$\mathcal{K}(\Delta\Phi) = \sum_{k=0}^\infty \mathcal{K}_k \cos\left(2\pi k\frac{\Delta\Phi}{\Phi_0}\right). \qquad (52)$$

The amplitudes $\mathcal{K}_k$ consist of two parts,[28] originating from the fluctuations of the diffusion coefficient, Fig. 5a, and the fluctuations of the electron density of states, Fig. 5b:

$$\mathcal{K}_k = \alpha\frac{e^4}{(2\pi\hbar)^2}\frac{D^2}{R^4}\int\frac{d\varepsilon d\varepsilon'}{16T^2}\cosh^{-2}\frac{\varepsilon}{2T}\cosh^{-2}\frac{\varepsilon'}{2T}$$
$$\times \sum_{i=1}^4\left(\mathcal{A}_k^{(i)}(\varepsilon,\varepsilon') + \mathcal{B}_k^{(i)}(\varepsilon,\varepsilon')\right), \qquad (53)$$

see Appendix D. Here α is a geometry dependent dimensionless factor of order of unity; functions $\mathcal{A}_k^{(i)}(\varepsilon,\varepsilon')$ and $\mathcal{B}_k^{(i)}(\varepsilon,\varepsilon')$ are defined by

$$\mathcal{A}_k^{(i)}(\varepsilon,\varepsilon')=2\int\left|\mathcal{D}_i\left(\varepsilon,\varepsilon',\frac{x}{R}\right)\right|^2\cos(2\pi kx)dx, \qquad (54a)$$

$$\mathcal{B}_k^{(i)}(\varepsilon,\varepsilon')=\int\mathrm{Re}\left\{\mathcal{D}_i^2\left(\varepsilon,\varepsilon',\frac{x}{R}\right)\right\}\cos(2\pi kx)dx \qquad (54b)$$

in terms of the diffuson components:

$$\mathcal{D}_i(\varepsilon,\varepsilon',q) = \frac{1}{i(\varepsilon-\varepsilon'+\varsigma_i\varepsilon_Z)+Dq^2+\gamma_\varphi+\Gamma_i(\varepsilon,\varepsilon')}. \qquad (55)$$

The index i in Eqs. (54) runs over different spin configurations of the diffuson $\mathcal{D}_i(\varepsilon,\varepsilon',q)$, which are related to the common classification of spin wave functions of two spin-1/2 particles in terms of triplet and singlet states, see Table II. In our notations, modes $i = 1, 2$ correspond to total spin $S_d = 1$ with non-zero projections on the magnetic field $m_d = \pm 1$, mode $i = 3$ represents total spin $S_d = 1$ with zero projection on the magnetic field, $m_d = 0$, and mode $i = 4$ is a singlet spin configuration. Energies $\varsigma_i\varepsilon_Z$ represent the effect of the Zeeman splitting of conduction electron states on various diffuson modes. Coefficients ς_i are given in Table II.

We explicitly separated two additive components, $\Gamma_i(\varepsilon,\varepsilon')$ and γ_φ, to the diffuson decay rate in Eq. (55). The component $\Gamma_i(\varepsilon,\varepsilon')$ corresponds to the contributions from spin-orbit interaction and scattering off magnetic impurities. The component γ_φ takes into account other processes, such as electron escape through the leads and scattering caused by electron-phonon and electron-electron interactions.[22]

A. Effect of classical spin impurities

Following the spirit of Section IV, we first analyze conductance fluctuations in the presence of classical $S \gg 1$ spins. This analysis allows us to explore the effect of the impurity spin polarization at various relations between parameters of the system. In the next subsection we calculate the amplitude of conductance correlation function, Eqs. (54), for spins $S \sim 1$ in a metal with strong spin orbit interaction.

We consider systems with fast spin relaxation time $\tau_T \ll \Delta t$, where Δt is the time lapse between the current
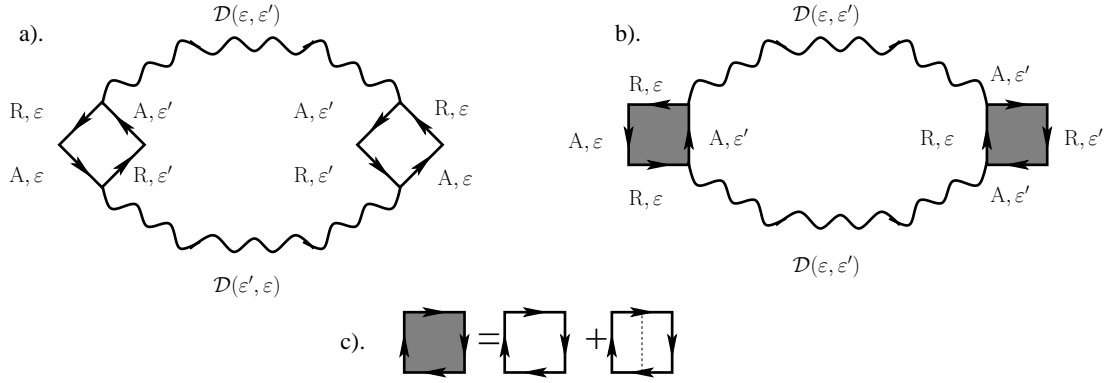
FIG. 5: Diagrams a) and b) give the main contribution to the conductance correlation function. Diagram a) originates from the fluctuations of the diffusion coefficient and decreases only as $1/T$ at high temperature. Diagram b) is referred to as the contribution from the fluctuations of the electron density of states and exponentially vanishes at high temperature, see[28]. The Hikami box (black square) is the sum of two diagrams, shown in Fig. c).

TABLE II: The decay rates of various components of the diffuson in a metal without spin orbit interaction. The components with $i = 1, 2, 3$ represent a spin states with total spin one, $S_\mathrm{d} = 1$. These spin configurations form a triplet in an isotropic system. Anisotropy due to the applied magnetic field lifts the degeneracy. The $i = 4$ component is a singlet spin configuration, $S_\mathrm{d} = 0$. The last column shows coefficients ς_i for the Zeeman splitting of conduction electrons.

i	$\lvert S_\mathrm{d}, m_\mathrm{d}\rangle$	$\Gamma_i(H)\tau_\mathrm{s}$	ς_i
1	$S_\mathrm{d} = 1, m_\mathrm{d} = +1$	$1 + \langle S_z\rangle^2/S^2$	$+1$
2	$S_\mathrm{d} = 1, m_\mathrm{d} = -1$	$1 + \langle S_z\rangle^2/S^2$	-1
3	$S_\mathrm{d} = 1, m_\mathrm{d} = \ 0$	$1 - \langle S_z\rangle^2/S^2$	0
4	$S_\mathrm{d} = 0, m_\mathrm{d} = \ 0$	$1 - \langle S_z\rangle^2/S^2$	0

measurements, see Eq. (50). In this case the solution of the diffuson equation, Eq. (A4), is described by the following energy-independent decay rates:

$$\Gamma_{1,2} = \left(1 + \langle S_z\rangle^2/S^2\right)\frac{1}{\tau_\mathrm{s}} + \frac{4}{3}\gamma_\mathrm{so}, \qquad (56a)$$

$$\Gamma_3 = \left(1 - \langle S_z\rangle^2/S^2\right)\frac{1}{\tau_\mathrm{s}} + \frac{4}{3}\gamma_\mathrm{so}, \qquad (56b)$$

$$\Gamma_4 = \left(1 - \langle S_z\rangle^2/S^2\right)\frac{1}{\tau_\mathrm{s}}. \qquad (56c)$$

Here $\langle S_z\rangle$ is defined by Eq. (29a) and depends only on the ratio of the magnetic field and temperature B/T.

The $i = 1, 2$ modes represent the interference of electron states with opposite spin orientations. The exchange field of magnetic impurities produces different (opposite) contributions to the phases of these two states. Because the phase contributions for different electron trajectories fluctuate, the interference of electron states with opposite spins is suppressed even if all spins of magnetic impurities are fully polarized. Thus, the polarization of impurity spins does not suppress the effect of electron phase relaxation due to the scattering off magnetic impurities for diffuson modes $i = 1, 2$. According to Eq. (56a), the

diffuson decay rates $\Gamma_{1,2}$ actually increase as the applied magnetic field increases.

On the other hand, the $i = 3, 4$ modes stand for the interference of two electron states with parallel spins. At strong magnetic field $B \gg T/g\mu_\mathrm{B}S$, when all spins are polarized, the scattering off magnetic impurities provides equal phase shifts to both states and does not affect diffuson relaxation rates $\Gamma_{3,4}$. That is why the contribution to $\Gamma_{3,4}$, caused by the scattering off magnetic impurities, vanishes as impurity spins become polarized, see Eqs. (56b) and (56c).

Substituting the diffuson decay times from Eqs. (56) into Eqs. (53) – (55), we can describe the harmonics of the conductance correlation function Eq. (52) at an arbitrary value of magnetic field, ranging from $B = 0$ to $B \gg T/g\mu_\mathrm{B}S$. To analyze this crossover, we first consider a metal without spin-orbit interaction, $\gamma_\mathrm{so} = 0$, and evaluate the integrals over energies ε and ε' in Eq. (53) for two limiting cases of low ($T \ll \Gamma_i + \gamma_\varphi$) or high temperature ($T \gg \Gamma_i + \gamma_\varphi$).

In the low temperature limit we have:[28]

$$\mathcal{K}_k = \frac{3\pi\alpha}{2}\frac{e^4}{(2\pi\hbar)^2}\sum_{i=1}^{4}\frac{L_i^3}{R^3}\left(2\pi k\frac{R}{L_i} + 1\right)e^{-2\pi kR/L_i} \quad (57)$$

with

$$L_i = \sqrt{\frac{D}{\Gamma_i + \gamma_\varphi}}. \qquad (58)$$

For simplicity, we omitted the Zeeman splitting of the conduction electron states, which actually modifies the $i = 1, 2$ terms in Eq. (57). We emphasize that at low temperature both diagrams in Fig. 5a and 5b contribute to the conductance correlation function.

In the limit of high temperature $T \gg \Gamma_i + \gamma_\varphi$, the contribution of the diagram in Fig. 5b is small as $\exp[-(2\pi)^{3/2}kR(T/D)^{1/2}]$ and may be disregarded. The contribution due to fluctuations of the diffusion coefficient, diagram in Fig. 5a, decays only as $1/T$ at high

temperature T:

$$\mathcal{K}_k = \alpha \frac{\pi^2}{3} \frac{e^4}{(2\pi\hbar)^2} \sum_{i=1}^{4} f\left(\frac{\varsigma_i \varepsilon_{\mathrm{Z}}}{2T}\right) \frac{L_T^2 L_i}{R^3} e^{-2\pi k R/L_i}, \quad (59)$$

where $L_T = \sqrt{D/T}$ is the thermal length, and coefficients ς_i are presented in Table II. The function

$$f(z) = 3\frac{z\cosh z - \sinh z}{\sinh^3 z} \quad (60)$$

takes into account the Zeeman splitting; $f(0) = 1$ and $f(z) \approx 12(z-1)e^{-2z}$ for $z \gg 1$.

Regardless of scattering off magnetic impurities, the Zeeman splitting destroys the contribution of modes $i = 1, 2$ to the conductance correlation function $\mathcal{K}$ at $\varepsilon_{\mathrm{Z}} \gg T$, see Eq. (59). From Eq. (59) we conclude that at strong magnetic field, $\varepsilon_{\mathrm{Z}} \gg T$ and $S\omega_S \gg T$, the amplitude of conductance fluctuations is no longer determined by the scattering rate off magnetic impurities, $1/\tau_{\mathrm{s}}$. Since modes $i = 1, 2$ are also suppressed at $\varepsilon_{\mathrm{Z}} \gg T$ even in a metal without spin impurities, we conclude that a strong magnetic field restores the amplitude of conductance fluctuations up to the amplitude in the same ring as if magnetic impurities were absent.

We also discuss another effect of the Zeeman splitting on the conductance correlation function. If the exchange constant J is antiferromagnetic, $J > 0$, the Zeeman splitting vanishes not only at $B = 0$ but also at some finite B, see Eq. (19), and thus produces another wrinkle in the dependence of $\mathcal{K}_k$ on the applied magnetic field. This non-monotonic behavior of the amplitude of the conductance oscillations is reminiscent to the reentrance effect in superconductors.[23]

To illustrate the effects of the Zeeman splitting ε_{Z}, we plot amplitude $\mathcal{K}_1$ of the principal harmonic, $k = 1$, as a function of B in Fig. 6. We choose the following values of the system parameters: $\gamma_\varphi \tau_{\mathrm{s}} = 2$ and $R = \sqrt{D/\gamma_\varphi}$. The dashed line corresponds to the case of equal g-factors for the conduction electrons and impurity spins, i.e., $\varepsilon_{\mathrm{Z}} = \omega_S$, and negligibly small exchange contribution in Eq. (19), $n_s J\langle S\rangle \ll \omega_S$. The limit of $\varepsilon_{\mathrm{Z}} \gg T$, when only terms $i = 3, 4$ in Eq. (59) survive, is represented by the solid line. Finally, the dotted line demonstrates the reentrance effect due to the antiferromagnetic impurities at some specific value of $n_{\mathrm{s}}J = 3.5T$ with T being the temperature.

Now we notice, see Eqs. (56), that the spin-orbit interaction suppresses the contribution to the amplitude of the conductance fluctuations, originating from the diffuson modes with $i = 1, 2, 3$ ($S_{\mathrm{d}} = 1$). In the limit of strong spin-orbit interaction ($\gamma_{\mathrm{so}} \gg D/R^2$) only the contribution from the singlet ($S_{\mathrm{d}} = 0$ and $i = 4$) mode survives:

$$\mathcal{K}_k = \alpha \frac{\pi^2}{3} \frac{e^4}{(2\pi\hbar)^2} \frac{L_T^2 L_4}{R^3} e^{-2\pi k R/L_4}, \quad (61)$$

with length L_4 defined by Eqs. (56c) and (58). The function $\mathcal{K}_k$ for a metal with strong spin orbit scattering
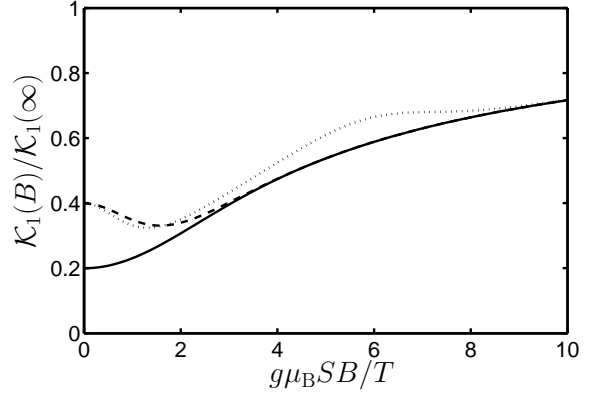

FIG. 6: Amplitude $\mathcal{K}_1(B)$ as a function of the applied magnetic field B for a metal with classical ($S \gg 1$) magnetic impurities in the absence of spin orbit interaction. A non-monotonic behavior of $\mathcal{K}_1$ due to the Zeeman splitting of the conduction electron states is illustrated for two cases: *i)* $\varepsilon_{\mathrm{Z}} = \omega_S$ (dashed line), *ii)* $\varepsilon_{\mathrm{Z}} = \omega_S - 7T\langle S_z\rangle$ (dotted line), here the second term represents the average exchange field $2n_{\mathrm{s}}J\langle S_z\rangle$ with $n_{\mathrm{s}}J = 3.5T$. The limit $\varepsilon_{\mathrm{Z}} \to \infty$ is presented by a solid line. We choose $\gamma_\varphi \tau_{\mathrm{s}} = 2$ and $R = \sqrt{D/\gamma_\varphi}$.

monotonically increases as a function of B/T, provided γ_φ remains constant. At strong magnetic field, $S\omega_S \gg T$, magnetic impurities do not affect the conductance fluctuations.

We emphasize that the non-monotonic behavior of the amplitudes $\mathcal{K}_k$ of the conductance correlation function originates from the diffuson modes with $i = 1, 2$, which do not contribute to the correlation function in metals with strong spin-orbit interaction. Thus, the monotonic or non-monotonic behavior of the amplitudes $\mathcal{K}_k$ distinguishes materials with or without spin orbit coupling.

B. Effect of quantum spin impurities

In this section we perform a quantum calculation to analyze the effect of polarization of impurity spins with $S \sim 1$, when the semiclassical description of electron scattering off magnetic impurities is not applicable. We consider metals with strong spin orbit interaction, $\gamma_{\mathrm{so}} \gg D/R^2$, so that only the singlet component of the diffuson survives. In this case the calculation of the diffuson self energy is similar to the calculation of the proper quantity for the Cooperon, see Sec. IV C. Details of calculations and the expression for the amplitude of the conductance fluctuations are presented in Appendix E.

Below we focus on the high temperature case, $T \gg \Gamma(\varepsilon, \varepsilon' \simeq T)$, which is reached at sufficiently strong magnetic field even if $\tau_{\mathrm{s}}T \ll 1$. At high temperature, the integral over the difference of ε and ε' converges fast, $|\varepsilon - \varepsilon'| \lesssim \Gamma(\varepsilon, \varepsilon' \simeq T)$. This observation allows us to per-
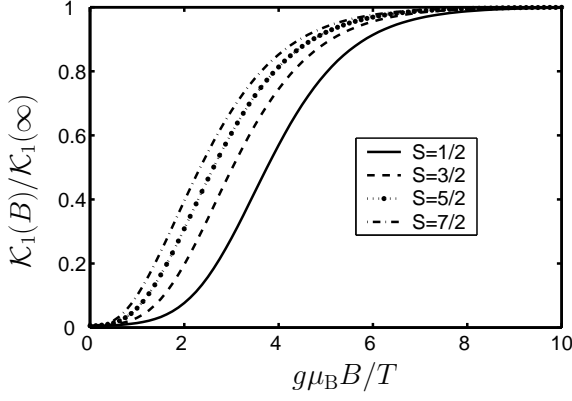
FIG. 7: The dependence of conductance oscillations $\mathcal{K}_1(B)$ as a function of the applied magnetic field B for four values of impurity spins: $S = 1/2$ (solid line), $S = 3/2$ (dashed line), $S = 5/2$ (dotted line), $S = 7/2$ (dash-dotted line). We choose $\gamma_\varphi \tau_s = 1/2$ and $R = \sqrt{D/\gamma_\varphi}$.

form integration over the energy difference. We obtain:

$$\mathcal{K}_k = \alpha \frac{\pi^2}{8} \frac{e^4}{(2\pi\hbar)^2} \frac{\sqrt{D} L_T^2}{R^3}$$
$$\times \int \frac{e^{-2\pi k R \sqrt{\Gamma(\varepsilon) + \gamma_\varphi}/\sqrt{D}}}{\sqrt{\Gamma(\varepsilon) + \gamma_\varphi}} \frac{d\varepsilon/T}{\cosh^4 \varepsilon/2T}, \quad (62)$$

with the dephasing rate in the form of

$$\Gamma(\varepsilon) = \left[1 - \frac{\langle \hat{S}_z \rangle^2 + \langle \hat{S}_z \rangle \tanh(\varepsilon + \omega_s)/2T}{S(S+1)} \right] \frac{1}{\tau_s}. \quad (63)$$

We represent the corresponding curves of $\mathcal{K}_1$ as a function of the applied magnetic field B in Fig. 7 for several values of S, assuming $\tau_s T \gg 1$ and for the following values of the system parameters: $\gamma_\varphi \tau_s = 1/2$ and $R = \sqrt{D/\gamma_\varphi}$. The shapes of the curves corresponding to various values of the impurity spin S are different from each other. Particularly, we conclude that the conductance fluctuations are faster restored by magnetic field in metals with a larger value of S. The fitting of experimental results by the curves $\mathcal{K}_k(B)$ given by Eq. (62), may provide the impurity spin parameters, such as its value S and the gyromagnetic factor g.

As the polarization $\langle S_z \rangle$ and the Zeeman splitting ω_S increase, the diffuson decay rate $\Gamma(\varepsilon \simeq T)$ to the second order in the exchange constant J becomes exponentially small, see Eq. (63). In this case we have to take into account higher-order contribution in J, which is represented by Eq. (47). The crossover from the phase relaxation rate in the form of Eq. (63) to the form of Eq. (47) takes place at magnetic field $B \sim B_*(T)$, see Eq. (8).

VI. CONCLUSIONS

The main question addressed in this paper, is how the polarization of magnetic impurities in a metal affects the interference contribution to its conductance. It is well-known that at weak magnetic field, which does not cause such polarization, the electron scattering off localized spins results in suppression of the weak localization correction to the conductivity and in suppression of the mesoscopic conductance fluctuations. If these interference contributions can be restored by an application of a stronger spin-polarizing field, then a transport measurement may serve as a test for the presence of magnetic impurities in a sample. This possibility was the main motivation of the presented study.

We obtained analytical results for the WL correction to the conductivity and for the amplitude of conductance fluctuations valid for an arbitrary magnetic field. We found that the conditions for the restoration of weak localization correction are quite stringent. The weak localization correction can be substantially enhanced by an application of a magnetic field only in samples of very small size (to avoid the orbital effect of the magnetic field) and made of a light-element material, to make the spin-orbit scattering negligible. If these conditions are met, the spin polarization may completely eliminate the effect of magnetic impurities. At intermediate fields B, the weak localization correction depends on the ratio B/T, and we find the corresponding crossover function spanning the full range of spin polarizations.

The effect of spin polarization on the mesoscopic conductance fluctuations is much more robust. The amplitude of fluctuations is restored if variations of a spin configuration in time are suppressed by the external magnetic field. At the same time, there is no detrimental orbital influence of the applied magnetic field on mesoscopic fluctuations. The spin-orbit interaction also does not spoil the effect of spin polarization on the amplitude of fluctuations. We evaluated the amplitude of the "h/e" Aharonov-Bohm oscillations of the conductance correlation function. In a metal with strong spin orbit interaction, the polarization of magnetic impurities restores this amplitude up to its value characteristic for the host material in the absence of magnetic impurities. We find the full crossover function, which describes the amplitude of fluctuations at arbitrary value of B/T. The use of our results for the analysis of transport measurements may, in principle, yield such characteristics of magnetic impurities as the values of their spin and g-factor, which are hard to access directly.

APPENDIX A: COOPERON AND DIFFUSON IN A METAL WITH CLASSICAL MAGNETIC IMPURITIES

In calculations of the weak localization correction to the conductivity of a metal with classical magnetic mo-

ments, we use the Cooperon which is formally defined as an average of electron Green's functions

$$
\mathcal{C}_{\gamma\delta}^{\alpha\beta}\left(\frac{t_1^+ - t_1^-}{2}, \frac{t_2^+ - t_2^-}{2}, t_1^+ + t_1^-; \mathbf{r}, \mathbf{r}'\right) \delta(t_1^+ + t_1^- - t_2^+ - t_2^-) \; = \; \langle G_{\alpha\beta}^{(\mathrm{R})}(t_1^+, t_2^+; \mathbf{r}, \mathbf{r}') G_{\gamma\delta}^{(\mathrm{A})}(t_1^-, t_2^-; \mathbf{r}, \mathbf{r}')\rangle. \quad \text{(A1)}
$$

After the standard procedure[1] we obtain the following expression for the Cooperon:

$$
\left[\left(\frac{\partial}{\partial\tau} + D\mathbf{q}^2\right)\hat{1} + \hat{\mathcal{H}}_0^{\mathcal{C}} + \hat{\mathcal{H}}_s^{\mathcal{C}}(\tau)\right]\hat{\mathcal{C}}(\tau, \tau', T; \mathbf{q}) = \hat{1}\delta(\tau - \tau'), \quad \text{(A2a)}
$$

where $\hat{\mathcal{H}}_0^{\mathcal{C}}$ is a matrix in spin space representing the Zeeman splitting of conduction electron states and spin-orbit interaction:

$$
\hat{\mathcal{H}}_0^{\mathcal{C}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2i\varepsilon_Z & 0 & 0 \\ 0 & 0 & -2i\varepsilon_Z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{2\gamma_{\mathrm{so}}}{3}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \text{(A2b)}
$$

and $\hat{\mathcal{H}}_{rms}^{\mathcal{C}}$ describes scattering off magnetic impurities:

$$
\hat{\mathcal{H}}_s^{\mathcal{C}}(\tau) = \frac{1}{\tau_s}\begin{pmatrix} (1 - \chi_{zz}(\tau)) & 0 & 0 & 0 \\ 0 & (1 + \chi_{zz}(\tau)) & -\chi_\perp(\tau) & 0 \\ 0 & -\chi_\perp^*(\tau) & (1 + \chi_{zz}(\tau)) & 0 \\ 0 & 0 & 0 & (1 - \chi_{zz}(\tau)) \end{pmatrix} \quad \text{(A2c)}
$$

The solution of Eqs. A2 yields the Cooperon in the form of Eq. (21).

The diffuson appears in this paper in calculations of the conductance correlation function and is defined as

$$
\mathcal{D}_{\gamma\delta}^{\alpha\beta}\left(\frac{t_1^+ + t_1^-}{2}, \frac{t_2^+ + t_2^-}{2}, t_1^+ - t_1^-; \mathbf{r}, \mathbf{r}'\right) \delta(t_1^+ - t_1^- - t_2^+ + t_2^-) \; = \; \langle G_{\alpha\beta}^{(\mathrm{R})}(t_1^+, t_2^+; \mathbf{r}, \mathbf{r}') G_{\delta\gamma}^{(\mathrm{A})}(t_2^-, t_1^-; \mathbf{r}', \mathbf{r})\rangle. \quad \text{(A3)}
$$

It satisfies the following equation:

$$
\left[\left(i\omega + D\mathbf{q}^2\right)\hat{1} + \hat{\mathcal{H}}_0^{\mathcal{D}} + \hat{\mathcal{H}}_s^{\mathcal{D}}(\tau)\right]\hat{\mathcal{D}}_{\mathbf{q}}(\omega, \tau) = \hat{1}, \quad \text{(A4a)}
$$

where the matrix $\hat{\mathcal{H}}_0^{\mathcal{D}}$ does not depend on time variable τ and describes the Zeeman splitting of conduction electron states and spin-orbit interaction:

$$
\hat{\mathcal{H}}_0^{\mathcal{D}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2i\varepsilon_Z & 0 & 0 \\ 0 & 0 & -2i\varepsilon_Z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{2\gamma_{\mathrm{so}}}{3}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad \text{(A4b)}
$$

and $\hat{\mathcal{H}}_s^{\mathcal{D}}(\tau)$ represents scattering off magnetic impurities:

$$
\hat{\mathcal{H}}_s^{\mathcal{D}}(\tau) = \frac{1}{\tau_s}\begin{pmatrix} (1 - \chi_{zz}(\tau)) & 0 & 0 & -\chi_\perp(\tau) \\ 0 & (1 + \chi_{zz}(\tau)) & 0 & 0 \\ 0 & 0 & (1 + \chi_{zz}(\tau)) & 0 \\ -\chi_\perp^*(\tau) & 0 & 0 & (1 - \chi_{zz}(\tau)) \end{pmatrix}. \quad \text{(A4c)}
$$

Solving Eqs. (A4), we obtain the diffuson in the form of Eq. (55).

APPENDIX B: COOPERON DECAY RATE DUE TO SCATTERING OFF QUANTUM MAGNETIC IMPURITIES

In this Appendix we derive the Cooperon decay rate in a metal with quantum $(S \sim 1)$ impurity spins. The
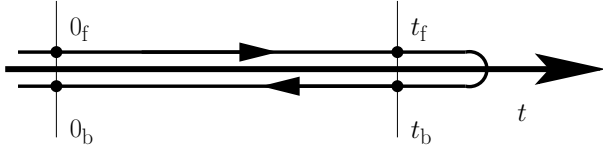
FIG. 8: The Keldysh contour allows to take into account time ordering of quantum operators. The time subscripts $t_{\mathrm{f,b}}$ mean that the operator is taken at time t at the forward (t_{f}) or backward (t_{b}) part of the Keldysh contour.

Hamiltonian of interaction of conduction electrons with magnetic impurities, Eq. (12), has the form:

$$H_{\mathrm{m}} = J \left(\hat{S}_z \hat{\sigma}_z + \hat{S}_- \hat{\sigma}_+ + \hat{S}_+ \hat{\sigma}_- \right) \qquad \text{(B1)}$$

where $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ are the Pauli matrices and $\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$. We notice that the first term in Eq. (B1) represent scattering without spin flip, while two other terms correspond to spin flip scattering.

We will follow the standard Keldysh formalism: first, we define electron and spin operators on the forward and backward parts of the Keldysh contour, see Fig. 8, and then perform rotation in the Keldysh space. As the result, we obtain the standard form of the electron Green's function in terms of the retarded $G^{(\mathrm{R})}(\varepsilon, \mathbf{p})$, advanced $G^{(\mathrm{A})}(\varepsilon, \mathbf{p})$ and Keldysh $G^{(\mathrm{K})}(\varepsilon, \mathbf{p})$ components. The spin operator in the rotated basis acquires the form

$$\hat{\mathcal{S}}_i(t) = \frac{1}{2} \left(\begin{array}{cc} \hat{S}_i^{\mathrm{s}}(t) & \hat{S}_i^{\mathrm{a}}(t) \\ \hat{S}_i^{\mathrm{a}}(t) & \hat{S}_i^{\mathrm{s}}(t) \end{array} \right), \qquad \text{(B2)}$$

where $\hat{S}_i^{\mathrm{s,a}}(t) = \hat{S}_i(t_{\mathrm{f}}) \pm \hat{S}_i(t_{\mathrm{b}})$. In our calculations of the impurity average conductivity, we work in the Born approximation, therefore the scattering is completely char-

acterized by the bilinear spin correlators:

$$\mathcal{L}_{ij}^{\mathrm{k}}(t) = \frac{1}{4} \langle \hat{S}_i^{\mathrm{s}}(t) \hat{S}_j^{\mathrm{s}}(t) \rangle, \qquad \text{(B3a)}$$

$$\mathcal{L}_{ij}^{\mathrm{r}}(t) = \frac{1}{4} \langle \hat{S}_i^{\mathrm{s}}(t) \hat{S}_j^{\mathrm{a}}(0) \rangle, \qquad \text{(B3b)}$$

$$\mathcal{L}_{ij}^{\mathrm{a}}(t) = \frac{1}{4} \langle \hat{S}_i^{\mathrm{a}}(t) \hat{S}_j^{\mathrm{s}}(0) \rangle, \qquad \text{(B3c)}$$

which are counterparts of the Keldysh Green's function of an interaction field, see[30].

For the Zeeman Hamiltonian $H_{\mathrm{spin}} = g\mu_{\mathrm{B}} B \hat{S}_z$, spin correlator components with $i = j = z$ have only the Keldysh element $\mathcal{L}_{zz}^{\mathrm{k}}(t)$:

$$\mathcal{L}_{zz}^{\mathrm{k}}(t) = \langle \hat{S}_z^{\mathrm{s}}(t) \hat{S}_z^{\mathrm{s}}(0) \rangle = \langle \hat{S}_z^2 \rangle, \qquad \text{(B4)}$$

and $\mathcal{L}_{zz}^{\mathrm{r,a}}(t)$ vanish due to the commutation relations of the spin operator along z axis and the Hamiltonian H_{spin}.

Now we present correlators of the spin components perpendicular to the magnetic field:

$$\mathcal{L}_{-+}^{\mathrm{a}}(t) = \theta(-t) e^{i\omega_{\mathrm{s}}t} \left\langle \hat{S}_z \right\rangle, \qquad \text{(B5a)}$$

$$\mathcal{L}_{-+}^{\mathrm{r}}(t) = -\theta(t) e^{i\omega_{\mathrm{s}}t} \left\langle \hat{S}_z \right\rangle, \qquad \text{(B5b)}$$

$$\mathcal{L}_{-+}^{\mathrm{k}}(t) = e^{i\omega_{\mathrm{s}}t} \left(S(S+1) - \langle \hat{S}_z^2 \rangle \right). \qquad \text{(B5c)}$$

Using the formal definitions of Eqs. (B4) and (B5), we calculate the Cooperon decay rate, which has the meaning of the Cooperon self energy, see Eq. (21). The diagrams which contribute to the decay rates of Cooperon modes $i = 1, 2$ are shown in Fig. 3a and 3b and correspond to the electron self-energy due to magnetic impurities:

$$\Sigma^{\mathrm{r}}(\varepsilon) = \int \frac{d\vec{p}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \left\{ \left(\mathcal{L}_{-+}^{\mathrm{k}}(\omega) + \mathcal{L}_{zz}^{\mathrm{k}}(\omega) \right) G^{\mathrm{r}}(\varepsilon + \omega, \mathbf{p}) + \left(\mathcal{L}_{-+}^{\mathrm{r}}(\omega) + \mathcal{L}_{zz}^{\mathrm{r}}(\omega) \right) G^{\mathrm{k}}(\varepsilon + \omega, \mathbf{p}) \right\}, \qquad \text{(B6a)}$$

$$\Sigma^{\mathrm{a}}(\varepsilon) = \int \frac{d\vec{p}}{(2\pi)^d} \int \frac{d\omega}{2\pi} \left\{ \left(\mathcal{L}_{-+}^{\mathrm{k}}(\omega) + \mathcal{L}_{zz}^{\mathrm{k}}(\omega) \right) G^{\mathrm{a}}(\varepsilon + \omega, \mathbf{p}) + \left(\mathcal{L}_{-+}^{\mathrm{a}}(\omega) + \mathcal{L}_{zz}^{\mathrm{a}}(\omega) \right) G^{\mathrm{k}}(\varepsilon + \omega, \mathbf{p}) \right\}. \qquad \text{(B6b)}$$

The elastic part of the self-energy is expressed only in terms of the retarded and advanced Green's function and is not affected by the electron distribution, while the inelastic part of the self energy contains also the Keldysh component of the electron Green's function. Consequently, for inelastic processes the electron distribution is important, since such processes should satisfy the fermion exclusion principle. The contribution to the Cooperon decay rate from diagrams in Figs. 5a and 5b is

$\Gamma_{\mathrm{ab}}(\varepsilon) = i n_{\mathrm{s}} J^2 [\Sigma^{\mathrm{a}}(\varepsilon) - \Sigma^{\mathrm{b}}(\varepsilon)]$ and can be written as

$$\Gamma_{\mathrm{ab}}(\varepsilon) = \frac{1}{\tau_{\mathrm{s}}} \left[1 - \int \tanh \frac{\varepsilon + \omega}{2T} \frac{\mathrm{Im}\left\{ \mathcal{L}_{-+}^{\mathrm{r}}(\omega) \right\}}{S(S+1)} \frac{d\omega}{2\pi} \right]. \qquad \text{(B7)}$$

The first term in Eq. (B7) originates from terms in Eqs. (B6b), containing the Keldysh components of the spin correlators $\mathcal{L}_{zz}^{\mathrm{k}}(\omega)$ and $\mathcal{L}_{-+}^{\mathrm{k}}(\omega)$. We used the property of these correlators $\mathcal{L}_{zz}^{\mathrm{k}}(t=0) + \mathcal{L}_{-+}^{\mathrm{k}}(t=0) = S(S+1)$, arising from their definitions, see Eqs. (B3). We observe that the first term in Eq. (B7) is the total electron

scattering rate on magnetic impurities $1/\tau_s$. The second term represents the rate of inelastic scattering processes, accompanied by the energy transfer. Particularly, this term explicitly contains the electron distribution function in the form $1 - 2n(\varepsilon) = \tanh \varepsilon/2T$, which takes into account the Pauli exclusion principle for electrons.

We identify the diagram in Fig. 3c with the vertex correction to the Cooperon self energy. For the Cooperon component with parallel electron spins, only the component of the scattering on magnetic impurities without spin flip remains:

$$\Gamma_c(\varepsilon) \;=\; \frac{1}{\tau_s} \frac{\langle \hat{S}_z^2 \rangle}{S(S+1)}. \tag{B8}$$

The decay rate of Cooperon modes $i = 1, 2$ is given by the sum of $\Gamma_{ab}(\varepsilon)$ and $\Gamma_c(\varepsilon)$:

$$\Gamma(\varepsilon) = \frac{1}{\tau_s} \left[1 - \frac{\langle \hat{S}_z^2 \rangle + \langle \hat{S}_z \rangle \tanh(\varepsilon + \omega_s)/2T}{S(S+1)} \right]. \tag{B9}$$

When deriving Eq. (B9) we assumed that the conduction electron spins and average impurity spins are parallel, so that $\omega_s > 0$ and $\langle \hat{S}_z \rangle > 0$. The opposite case is also described by Eq. (B9) with $\omega_s < 0$ and $\langle \hat{S}_z \rangle < 0$. Equation (B9) determines the energy-dependent phase relaxation rate in the WL correction to conductivity, see Eq. (30).

APPENDIX C: THE J^4-ORDER CONTRIBUTION TO THE ELECTRON SELF ENERGY

In this Appendix we derive the correction to the Cooperon self energy to the fourth order in the exchange constant J. For this purpose we will use the rules of the Keldysh diagrammatic technique, outlined in Section IV C.

The contribution to the Cooperon self energy is shown in Fig. 4. The electron-hole loops can be treated as the self energy $\hat{\varrho}$ of the spin-spin Keldysh Green's function, responsible for the spin relaxation as a result of the electron inelastic scattering. The imaginary part of this electron-hole loop determines the Korringa spin relaxation rate.[7] In the Keldysh formalism, we can distinguish three components of the self energy, related to the advanced, retarded and Keldysh elements of the spin-spin Green's function.

We have for the difference of retarded and advanced components of the spin-spin self energy the following expression

$$\varrho^r(\omega) - \varrho^a(\omega) = 2\pi \nu^2 J^2 \omega, \tag{C1}$$

and the Keldysh component is

$$\varrho^k(\omega) = \coth \frac{\omega}{2T} \left(\varrho^r(\omega) - \varrho^a(\omega) \right). \tag{C2}$$

This result is of no surprise. Indeed, the self energy is not related to the type of interaction, since it is uniquely determined by the electron-hole loop, rather than the interaction lines.

Here we are interested in the strong magnetic field limit, $\omega_S \gg T$, so that only two impurity spin states with $S_z = S$ and $S_z = S - 1$ are involved. Under this condition, the diagrams in Fig. 4 can be represented in terms of the modified spin-spin correlators, which to the second order in the exchange constant $\nu^2 J^2$ have the form:

$$\mathcal{L}_1^{r,a}(\omega) \;=\; \mathcal{L}_{+-}^{(0)r,a}(\omega) \varrho^{r,a}(\omega) \mathcal{L}_{+-}^{(0)r,a}(\omega) \tag{C3}$$

$$\mathcal{L}_1^k(\omega) \;=\; \mathcal{L}_{+-}^{(0)r}(\omega) \varrho^k(\omega) \mathcal{L}_{+-}^{(0)a}(\omega), \tag{C4}$$

where $\mathcal{L}_1^{(0)r,a}(\omega) = \langle S \rangle/(\omega - \omega_S \pm i0)$ are the bare spin-spin correlation functions, defined by Eq. (B5). Using this notations, we reduce diagrams in Fig. 4 to diagrams in Figs. 3a and 3b. Then, after the standard calculations, we obtain

$$\Gamma_4(\varepsilon) = 2\pi n_s \nu^3 J^4 S^2 \frac{\pi^2 T^2 + \varepsilon^2}{\omega_S^2}. \tag{C5}$$

Using the definition of τ_s, we rewrite Eq. (C5) in the form of Eq. (47).

APPENDIX D: CONDUCTANCE CORRELATION FUNCTION

In this Appendix we outline the calculation of the conductance correlation function $\mathcal{K}(\Delta\Phi)$, see Eq. (52). The contributions to $\mathcal{K}(\Delta\Phi)$ originate from two diagrams, shown in Fig. 5. We have

$$\mathcal{K}(\Delta\Phi) \;=\; \mathcal{K}^{(a)}(\Delta\Phi) + \mathcal{K}^{(b)}(\Delta\Phi), \tag{D1}$$

where the first term $\mathcal{K}^{(a)}(\Delta\Phi)$ is the contribution from fluctuations of the diffusion coefficient

$$\mathcal{K}^{(a)}(\Delta\Phi) = \alpha \frac{e^4}{(2\pi\hbar)^2} \frac{D^2}{R^4} \int \frac{d\varepsilon d\varepsilon'}{16T^2} \frac{1}{\cosh^2 \varepsilon/2T} \frac{1}{\cosh^2 \varepsilon'/2T}$$
$$\times 2 \sum_{n=-\infty}^{+\infty} \sum_i \left| \mathcal{D}_i \left(\varepsilon, \varepsilon', q_n - \frac{\Delta\Phi}{R\Phi_0} \right) \right|^2, \tag{D2}$$

and second term in Eq. (D1) originates from fluctuations of the electron density of states:

$$\mathcal{K}^{(b)}(\Delta\Phi) = \alpha \frac{e^4}{(2\pi\hbar)^2} \frac{D^2}{R^4} \int \frac{d\varepsilon d\varepsilon'}{16T^2} \frac{1}{\cosh^2 \varepsilon/2T} \frac{1}{\cosh^2 \varepsilon'/2T}$$
$$\times \sum_{n=-\infty}^{+\infty} \sum_i \text{Re}\left\{ \mathcal{D}_i^2 \left(\varepsilon, \varepsilon', q_n - \frac{\Delta\Phi}{R\Phi_0} \right) \right\}. \tag{D3}$$

Here $\mathcal{D}_i(\varepsilon, \varepsilon', q)$ is the ith diffuson component and the sum over n runs through all discreet values of momentum $q_n = n/R$. We apply the Poisson summation formula

$$\sum_{n=-\infty}^{+\infty} f(n) = \sum_{k=-\infty}^{\infty} \int f(n) e^{2\pi i k n} dn$$

to Eqs. (D2) and (D3) and obtain Eq. (52) with amplitudes $\mathcal{K}_k$ defined by Eqs. (53) and (54).

APPENDIX E: DIFFUSON DECAY RATE DUE TO SCATTERING OFF QUANTUM MAGNETIC IMPURITIES

We consider the singlet contribution to the conductance fluctuations in the limit of short spin relaxation time $\tau_{\rm T}$, so that the components of spin perpendicular to magnetic field are not correlated at the characteristic time Δt between conductance measurements, $f_\perp(\Delta t) \to 0$ in Eqs. (16). In this case the diffuson self-energy is defined by diagrams similar to ones shown in Fig. (3), but with the opposite direction of the advanced Green's function.

The diffuson self energy consists of three terms; two of them coincide with the self energies $\Sigma^{\rm r}(\varepsilon)$ and $\Sigma^{\rm a}(\varepsilon)$ of retarded and advanced electron Green functions, respectively, and the third term is the vertex correction $\Sigma^{\rm v}(\varepsilon)$. Using the formalism, presented in Appendix B,

we obtain the following expression for the self energy of electron Green functions, see Eqs. (B6b):

$$\Sigma^{\rm r,a}(\varepsilon) = \frac{1}{2\tau_{\rm s}} \left[1 - \frac{\langle \hat{S}_z \rangle^2 + \langle \hat{S}_z \rangle \tanh(\varepsilon + \omega_{\rm s})/2T}{S(S+1)} \right. $$
$$\left. \pm\ i \frac{\langle \hat{S}_z \rangle}{S(S+1)} \int \frac{\tanh(\varepsilon + \omega)/2T}{\omega + \omega_{\rm s}} \frac{d\omega}{2\pi} \right]. \quad (E1)$$

The vertex correction to the diffuson originates from the impurity spin correlator, taken at time difference Δt, see Eq. (50). Since we assume that the impurity spin relaxation time is much smaller than the delay time Δt, the spin correlator is $\chi_z = \langle S_z \rangle^2$ and we have

$$\Sigma^{\rm v}(\varepsilon) = \frac{1}{\tau_{\rm s}} \langle S_z \rangle^2. \quad (E2)$$

Combining the electron self energy part and the vertex correction given by Eqs. (E1) and (E2), we obtain the diffuson self energy in the form:

$$\Gamma(\varepsilon, \varepsilon') = \frac{1}{\tau_{\rm s}} \left[1 - \frac{\langle \hat{S}_z \rangle^2}{S(S+1)} - \frac{\langle \hat{S}_z \rangle}{2S(S+1)} \left(\tanh \frac{\varepsilon + \omega_{\rm s}}{2T} + \tanh \frac{\varepsilon' + \omega_{\rm s}}{2T} \right) \right.$$
$$\left. +\ i\langle \hat{S}_z \rangle \int \frac{\tanh(\varepsilon + \omega)/2T - \tanh(\varepsilon' + \omega)/2T}{\omega + \omega_{\rm s}} \frac{d\omega}{2\pi} \right]. \quad (E3)$$

For the calculation of the conductance fluctuations, we have to consider a finite difference between the energies ε and ε', transferred along the advanced and retarded Green's functions. The last term in Eq. (E3) originates from the real part of the electron self energy and vanishes if $\varepsilon = \varepsilon'$. This term represent renormalization of the electron density of states.

The amplitude of the kth harmonic of the conductance correlation function is expressed in terms of $\mathcal{A}_k^{(4)}$ and $\mathcal{B}_k^{(4)}$, see Eqs. (53) and (54):

$$\mathcal{A}_k^{(4)} = \sqrt{\frac{D}{R^2}} \frac{2\pi}{\varepsilon - \varepsilon' + {\rm Im}\Gamma(\varepsilon, \varepsilon')} {\rm Im} \left\{ \frac{\exp\left(-2\pi k R \sqrt{\gamma_\varphi + \Gamma(\varepsilon, \varepsilon') + i(\varepsilon - \varepsilon')}/\sqrt{D} \right)}{\sqrt{\gamma_\varphi + \Gamma(\varepsilon, \varepsilon') + i(\varepsilon - \varepsilon')}} \right\} \quad (E4a)$$

$$\mathcal{B}_k^{(4)} = \sqrt{\frac{D\pi^3}{8R^2}} {\rm Re} \left\{ \left(1 + 2\pi k \frac{R\sqrt{D}}{\sqrt{\gamma_\varphi + \Gamma(\varepsilon, \varepsilon') + i(\varepsilon - \varepsilon')}} \right) \exp\left(-2\pi k R \frac{\sqrt{\gamma_\varphi + \Gamma(\varepsilon, \varepsilon') + i(\varepsilon - \varepsilon')}}{\sqrt{D}} \right) \right\}. \quad (E4b)$$

At high temperature $T\tau_{\rm s} \gg 1$ integral in Eq. (53) over $\varepsilon - \varepsilon'$ converges at $|\varepsilon - \varepsilon'| \lesssim 1/\tau_{\rm s}$, and we obtain Eqs. (62) and (63).

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